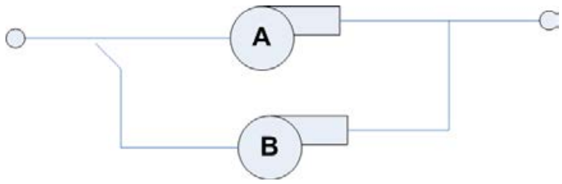
PROBLEM 4

We shall evaluate the maintenance of a pump system. The pump system comprises two pumps, an active pump (A) and a stand-by pump (B) as shown in the figure below.



If the active pump fails, an attempt is made to start the stand-by pump. The probability that the stand-by pump will start is considered to be p = 90%. The stand-by pump has only 50% capacity of the active pump. We assume that the active pump has an increasing hazard rate, $z(t)$. We assume that the stand-by pump will run for a short period of time upon failure of the active pump. In this period we assume that the probability of failure of the stand-by pump equals zero.

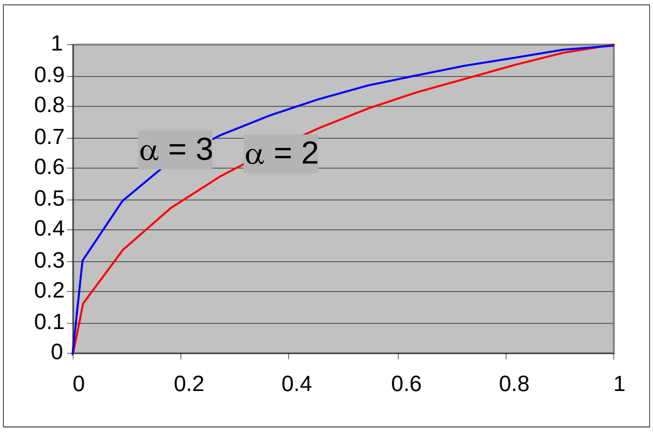
Preventive maintenance of the active pump is periodical replacement of wear parts. We do not consider maintenance of the stand-by pump in this problem. The relevant quantities to consider in the optimization are as follows.

Parameter	Value	Explanation
$MTTF_A$	6 000	Mean time to failure for active pump (A) in hours (without preventive maintenance).
p	0.9	The probability that the standby pump will start upon a demand, i.e., when an attempt to start the standby pump is made
α	3	Aging parameter for the active pump.
C_{PM}	1 000	Cost in NOKs of execution of a PM activity on the active pump.
C_{CM}	5 000	Cost in NOKs of repairing the active pump upon failure. We do not consider maintenance cost of the passive pump.
MDT	4	Mean down time after failure of the active pump.
C_U	15 000	Cost in NOKs of one hour loss of production, i.e., when none of the pumps are functioning. If the passive pump (B) is operating, the production loss is 7 500 NOKs due to lower capacity of this pump.

In this problem you may approximate the effective failure rate for the active pump by the following formula:

$$\lambda_E(\tau) = \left(\frac{\Gamma(1+1/\alpha)}{\text{MTTF}}\right)^{\alpha} \tau^{\alpha-1}$$

where $\Gamma(\cdot)$ denotes the gamma function, and $\Gamma(4/3) \approx 0.893$. You may also need the TTT transform for the Weibull distribution for two values of α:



a) Write down an expression for the expected cost per unit time as a function of τ, and find the optimal interval. Hint. Calculate the cost for $\tau = 1$ month, 2 months and so on. One month = 730 hours.

b) Failure data for a similar pump as the active pump has been collected. This pump has been operated on a "Run to failure" basis due to less critical production. Failure times (in hours) are 8800, 4000, 7100, 2500, 6500, 7200, 6600, 1800, 7200, 9700, and 7100. Construct a TTT plot for the data, and give a rough estimate for the parameters. Comment on the result.

c) In points a) and b) we have assumed that failure progression could not be observed prior to a failure of the active pump. There are several symptoms which might be observed, such as temperature increase, vibration and noise. In this problem we will assume that we may apply the so-called PF-model. We assume that "Mean time to a potential failure" (MTTPF) equals 4 000 hours. We assume that potential failures are caused by external shocks so that time to a potential failure for an as good as new pump is exponential distributed. Further we assume that the mean value of the PF interval equals 2 000 hours. The standard deviation in the PF interval equals 500 hours.

d) Explain the main features of the PF model. Assume first that no maintenance is conducted. Find the Mean Time To Failure in this situation. Let $Q_0(\tau)$ be the probability that a potential failure is not revealed by the inspection program given the inspection interval equals τ. Further let f_P = 1/MTTPF be the rate of potential failures. Give arguments why the effective failure rate in this situation may be written as $\lambda_E(\tau) \approx f_P\, Q_0(\tau)$.

We assume that a walk around check would be appropriate to detect the outset of a failure, i.e., a potential failure. The probability that a potential failure is not revealed by such a check is given by $q = 0.1$. The cost of a walk around is $C_I = 50$ NOKs. If a potential failure is revealed during testing the cost of replacing the active pump is C_{PM} as given in problem 4. Write down the cost model to use in the optimization of the optimum interval for walk around checks, and find the optimal interval. You may use the following approximation: $Q_0(\tau) = Q_0(\tau | E(T_{\mathrm{PF}}) = 2000, \mathrm{SD}(T_{\mathrm{PF}}) = 500, q = 0.1) \approx 7 \cdot 10^{-08}\tau^2 - 5 \cdot 10^{-05}\tau + 0.0089$. Compare the cost with the situation in Problems a) and b). Hint: Calculate the cost per unit time for $\tau = 5$, 6 and 7 weeks respectively. One week = 24·7 hours = 168 hours.

Solution

Problem 4

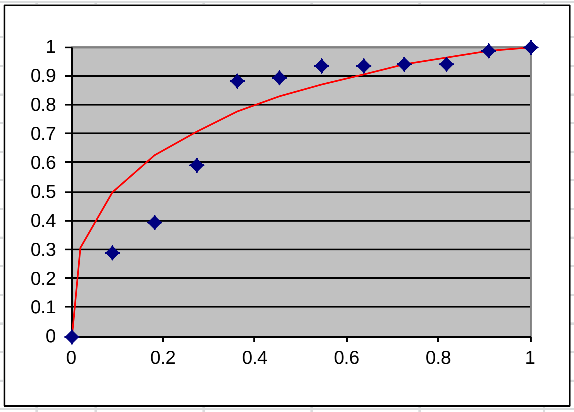
a) $C(\tau_A) = C_{\mathrm{PM}}/\tau_A + \lambda_E(\tau_A) \times [C_{\mathrm{CM}} + (1\text{-}p) \times C_U \times \mathrm{MDT} + 0.5p \times C_U \times \mathrm{MDT}]$

τ_A	$C(\tau_A)$
730	1.436621
1460	0.951961
2190	1.057438
2920	1.410586
3650	1.94291
4380	2.63158

Minimum at 1460 hours = 2 months. Analytical solution gives minimum at approximately 1590 hours.

b)

$T_{(i)}$	$\Sigma T_{(i)}$	$\Sigma T_{(i)+}(n\text{-}i)T_{(i)}$	i/n	TTT	Transform
1800	1800.00	19800	0.09	0.29	0.50
2500	4300.00	26800	0.18	0.39	0.62
4000	8300.00	40300	0.27	0.59	0.71
6500	14800.00	60300	0.36	0.88	0.77
6600	21400.00	61000	0.45	0.89	0.83
7100	28500.00	64000	0.55	0.93	0.87
7100	35600.00	64000	0.64	0.93	0.91
7200	42800.00	64400	0.73	0.94	0.94
7200	50000.00	64400	0.82	0.94	0.96
8800	58800.00	67600	0.91	0.99	0.98
9700	68500.00	68500	1.00	1.00	1.00



MTTF= 68500.00 /11= 6227

Red curve: α = 3

Results are reasonably OK. Need not to update the analysis.

c)

See course compendium. The "P" in the PF model is the point of time where the outset of a failure for the first time is observable by available inspection methods. The "F" is the time when the component fails. The time between the "P" and the "F" is denoted the PF interval.

Without maintenance, the MTTF is given by: MTTF = MTTPF + Mean value of the PF interval = 4000 + 2000 = 6000 as in problem 4. If f_P is the rate of potential failures, and $Q_0(\tau)$ is the probability that a potential failure is not revealed by the inspection program, then every $Q_0(\tau)^{-1}$ potential failure will go to a real failure. Hence $\lambda_E(\tau) \approx f_P\, Q_0(\tau)$ will be a reasonable approximation for the effective failure rate. In reality the effective failure rate will be slightly lower because in average each period will slightly longer than $1/f_P$ because we will not replace before the next inspection.

d)

$C(\tau_I) = C_I/\tau_I + f_P Q_0(\tau_I)\times[C_{CM} + (1-p) \times C_U \times \text{MDT} + 0.5p \times C_U \times \text{MDT}] + f_P(1-Q_0(\tau_I))\times C_{PM}$

In addition to pay for failures, we also need to pay for repairing the pump if a potential failure is revealed, i.e., with rate $f_P(1-Q_0(\tau_I))$.

τ_I	$C(\tau_I)$
168	3.101191
336	1.61345
504	1.122186
672	0.892565
840	0.783678
1008	0.746099
1176	0.754077
1344	0.791191

Minimum at 1008 hours = 6 weeks